Exam 2007

a current varies with time as shown in fig
sketch the variation of the voltage produced by this current
flowing in an initally uncharged capacitor

[1]

$C = 10^{-6}$ f

$i(t)$

\+ $v(t)$ −

$v(0) = 0$

$i(t) = C \frac{dv(t)}{dt}$

$v(t) = \frac{1}{C} \int i(t).dt$

$= \frac{1}{10^{-6}} \int i(t).dt \quad = 10^{6} \int i(t)$

$$i(t) = \begin{cases} 0 & t < 0 \\ -\frac{18}{3}t + 12 \;\; (\text{written over } \tfrac{-6}{1}) & 0 \leq t \leq 3 \\ \frac{6}{2}t + A \;\; (A = -15) & 3 \leq t \leq 5 \\ 0 & t > 5 \end{cases}$$

$i(3) = -18 + 12 = -6$

$\therefore -6 = \frac{6}{2} * 3 + A \longrightarrow A = -15$

$$v(t) = \begin{cases} 0 & t < 0 \\ -3t^2 + 12t + B & 0 \leq t \leq 3 \\ \frac{3}{2}t^2 - 15t + C & 3 \leq t \leq 5 \\ D & t \geq 5 \end{cases}$$

$v(0) = \text{Zero}$

$\therefore 0 = 0 + 0 + B \longrightarrow B = 0$

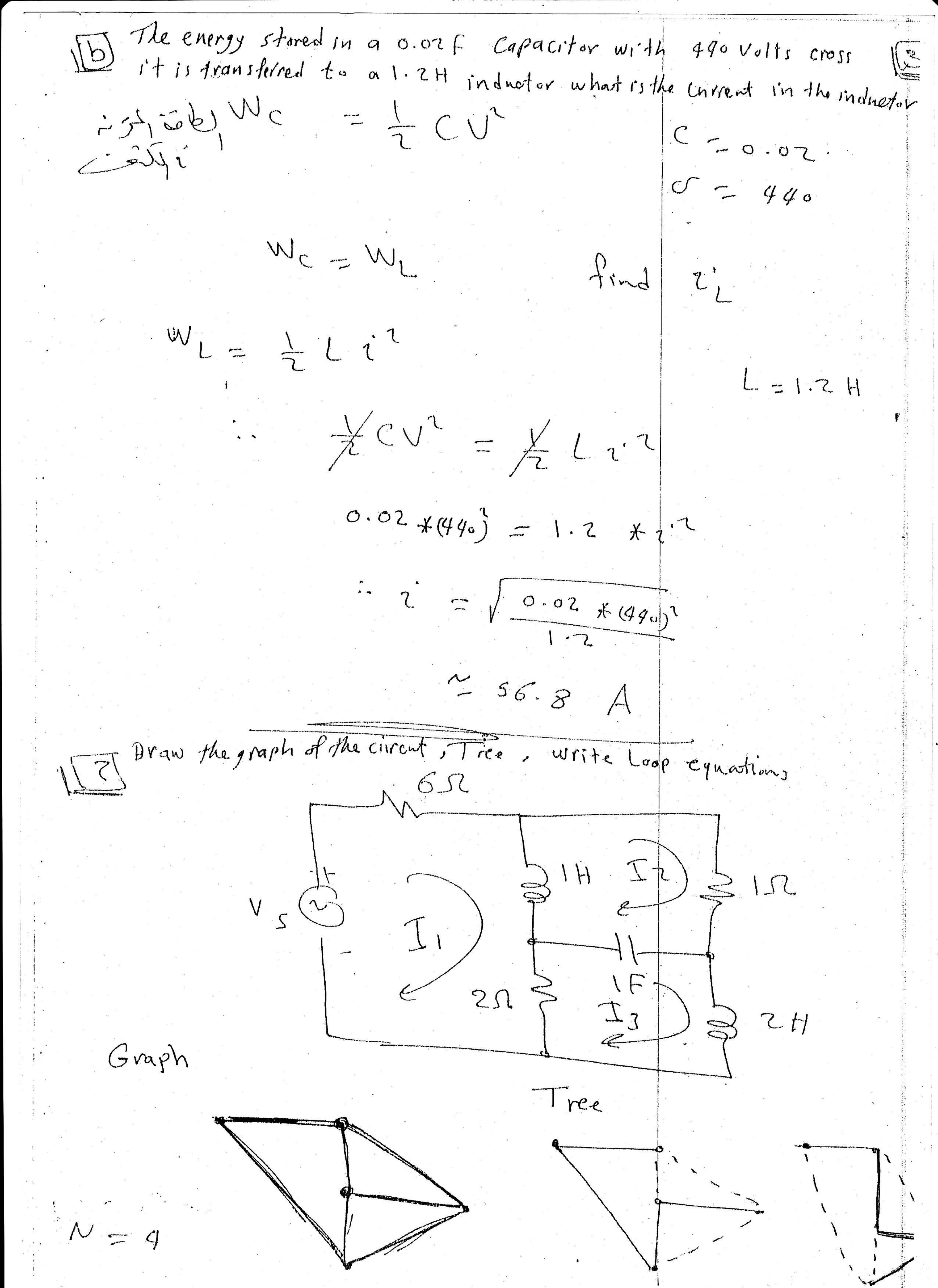
$v(3) = -27 + 36 + 0 \longrightarrow v(3) = 9$

$\therefore 9 = \frac{3}{2}(3)^2 - 15 * 3 + C \longrightarrow C = 40.5$

$v(5) = \frac{3}{2}(5)^2 - 15 * 5 + 40.5 = 3$

$\therefore D = 3$

$$v(t) = \begin{cases} 0 & t < 0 \\ -3t^2 + 12t + 0 & 0 \leq t \leq 3 \\ \frac{3}{2}t^2 - 15t + 40.5 & 3 \leq t \leq 5 \\ 3 & t \geq 5 \end{cases}$$



b The energy stored in a 0.02 F capacitor with 440 volts cross it is transferred to a 1.2 H inductor what is the current in the inductor

الطاقة المخزنة بالمكثف $W_C = \frac{1}{2} C V^2$

$C = 0.02$

$v = 440$

$W_C = W_L$

find i_L

$W_L = \frac{1}{2} L i^2$

$L = 1.2\ H$

$\therefore \frac{1}{2} C V^2 = \frac{1}{2} L i^2$

$0.02 * (440)^2 = 1.2 * i^2$

$\therefore i = \sqrt{\frac{0.02 * (440)^2}{1.2}}$

$\simeq 56.8\ A$

2 Draw the graph of the circuit, Tree, write Loop equations

Graph

Tree

$N = 4$

K.V.L for Loop 1

$$V_S = 6 I_1 + 1 \frac{d(I_1 - I_2)}{dt} + 2 (I_1 - I_3)$$

K.V.L for Loop 2

$$0 = \frac{d(I_2 - I_1)}{dt} + \frac{1}{1}\int (I_2 - I_3) + 1 I_2$$

K.V.L for Loop 3

$$0 = 2 (I_3 - I_1) + 1 \int (I_3 - I_2) + 2 \frac{d I_3}{dt}$$

3

Circuit: 9 V source, X, R_1, I_X, Loop 1, I_y, Y, I, R_2

if

$V_X = 4$ Volts

$I_X = 1.5 mA$

$V_y = 2V$

$I_y = 1 mA$.

Find R_1, R_2

Solution

$I_X = 1.5 mA = I_{R_1}$

$I_y = 1 mA$

$\therefore I = I_X - I_y = 1.5 mA - 1 mA = 0.5 mA$

$V_y = V_{R_2}$

$$\therefore R_2 = \frac{V_y}{I} = \frac{2}{0.5\,mA} = 4\,k\Omega.$$

K.V.L for Loop 1

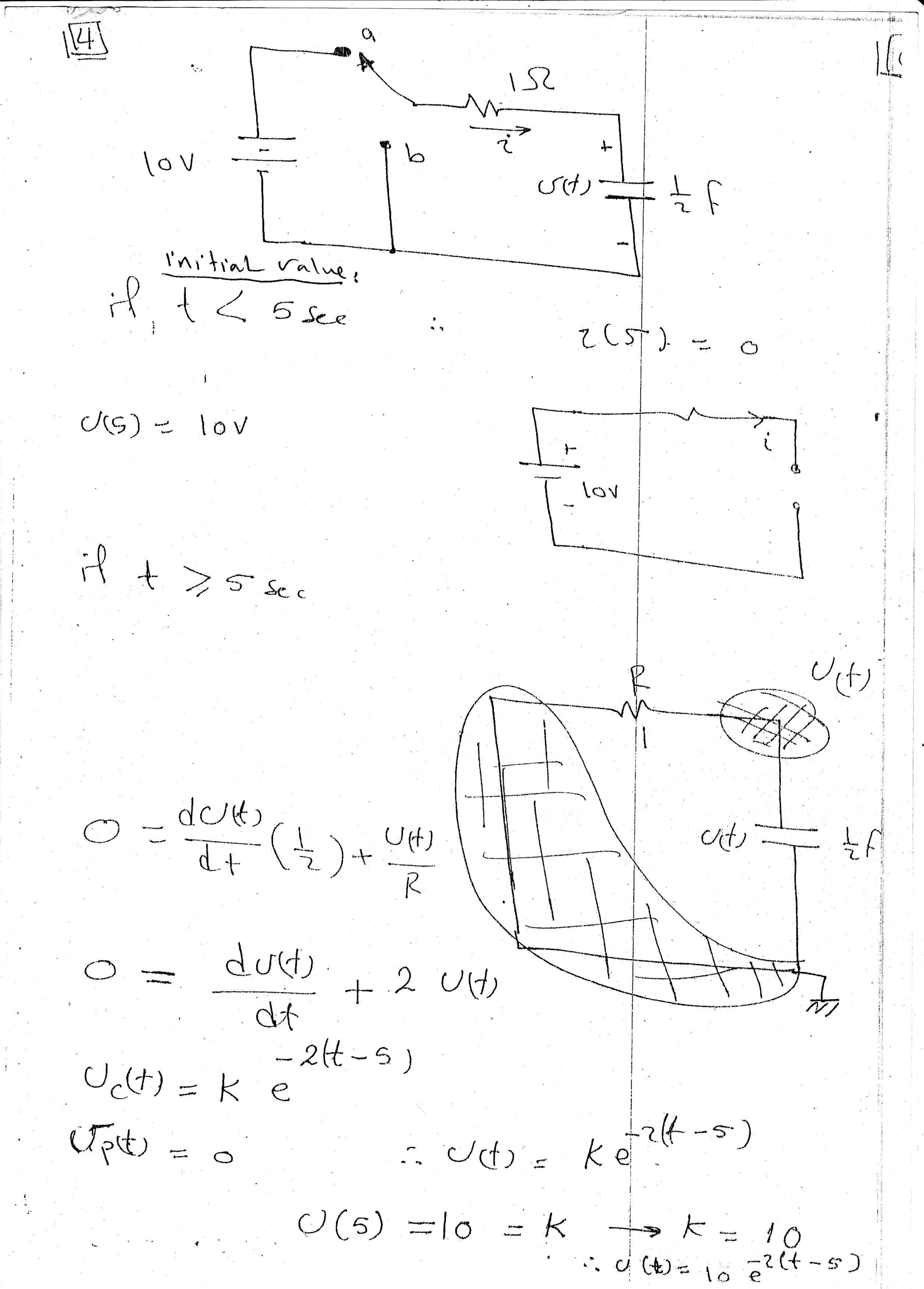
$$-9 + V_X + \underbrace{I_X R_1}_{V_{R_1}} + V_y = 0$$

$$\therefore V_{R_1} = 9 - V_X - V_y$$

$$= 9 - 4 - 2 = 3\,V$$

$$\therefore R_1 = \frac{3}{1.5\,mA} = 2\,k\Omega.$$

4) if the switch is closed to a at $t = 0$ and is transferred to b at $t = 5$ sec sketch the variation of the current i with time for $0 < t < 10$ sec.



initial value:

if $t < 5$ sec ∴ $i(5^-) = 0$

$v(5) = 10V$

if $t \geq 5$ sec

$$0 = \frac{dv(t)}{dt}\left(\frac{1}{2}\right) + \frac{v(t)}{R}$$

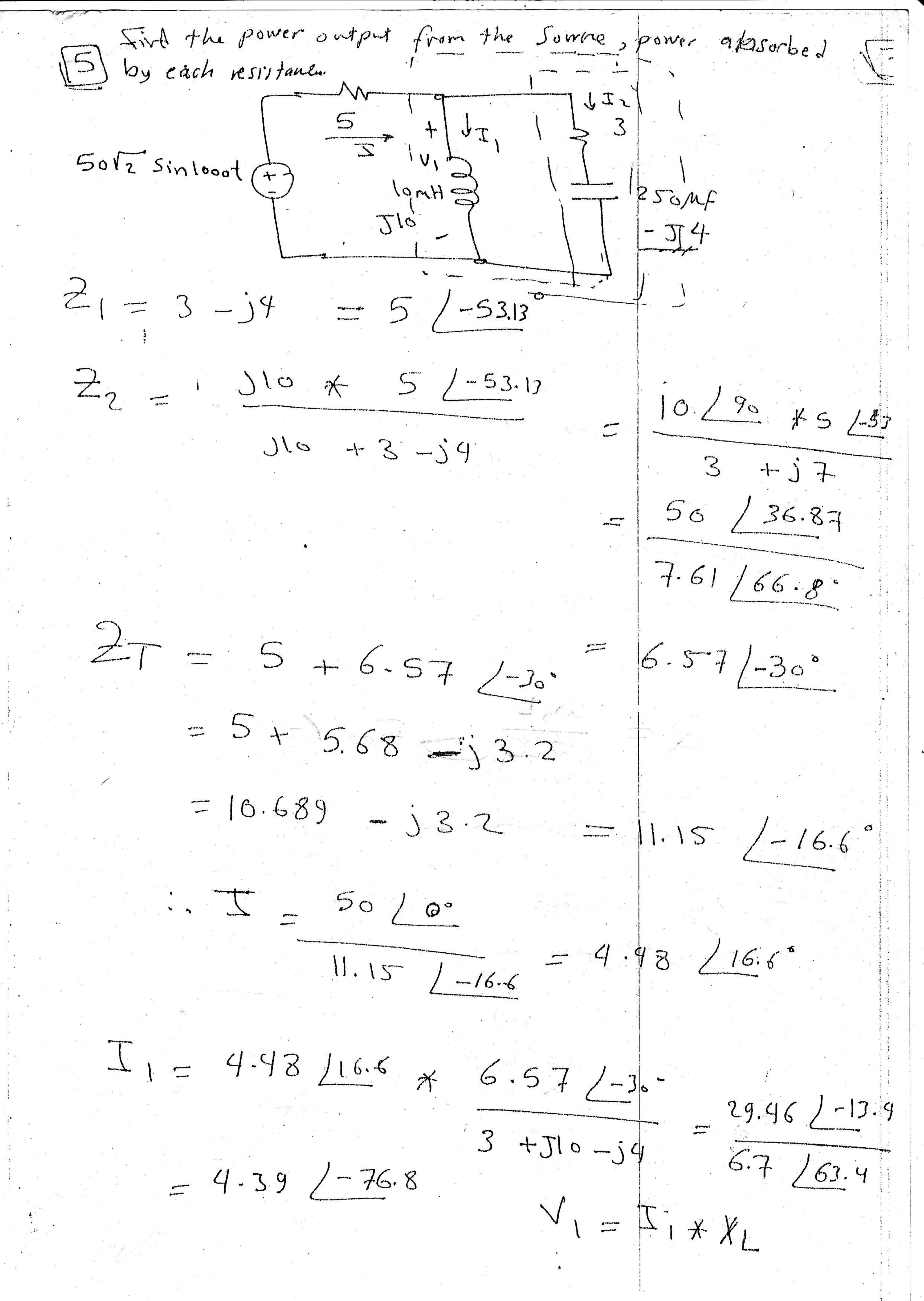
$$0 = \frac{dv(t)}{dt} + 2\,v(t)$$

$$v_c(t) = K\,e^{-2(t-5)}$$

$v_p(t) = 0$ ∴ $v(t) = K e^{-2(t-5)}$

$v(5) = 10 = K \rightarrow K = 10$

∴ $v(t) = 10\,e^{-2(t-5)}$



5 Find the power output from the Source, power absorbed by each resistance.

$$Z_1 = 3 - j4 = 5\angle -53.13^\circ$$

$$Z_2 = \frac{j10 * 5\angle -53.13}{j10 + 3 - j4} = \frac{10\angle 90 * 5\angle -53}{3 + j7} = \frac{50\angle 36.87}{7.61\angle 66.8^\circ} = 6.57\angle -30^\circ$$

$$Z_T = 5 + 6.57\angle -30^\circ$$

$$= 5 + 5.68 - j3.2$$

$$= 10.689 - j3.2 = 11.15\angle -16.6^\circ$$

$$\therefore I = \frac{50\angle 0^\circ}{11.15\angle -16.6} = 4.48\angle 16.6^\circ$$

$$I_1 = 4.48\angle 16.6 * \frac{6.57\angle -30^\circ}{3 + j10 - j4} = \frac{29.46\angle -13.4}{6.7\angle 63.4}$$

$$= 4.39\angle -76.8$$

$$V_1 = I_1 * X_L$$

$\therefore V_1 = 4.39 \angle -76.8 * 10 \angle 90°$

$= 43.9 \angle 13.2$

$S_1 = I_1^* V_1$

$= 4.39 \angle 76.8 * 43.9 \angle 13.9$

$= 192.7 \angle 89.9$

$S_1 = 0 + j192.7 = 192.7 \angle 90°$

$P_R = |I|^2 * R$

$= (4.98)^2 * 5 = 100.35$ watt.

نفس الطريقة بنحسب القدرة في العنصر

$R = 3\Omega$

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متروك

للطالب

Find $i(t)$

2 Sin (3t + 45)°
$I_s(t)$
$i(t)$
2
2 H
$\frac{1}{12}$ f
3 Sin (2t + ...
$V_s(t)$

Find $i(t)$

Solution
Using Super position:

$$i(t) = i_1(t) + i_2(t)$$

2
$\frac{2}{\sqrt{2}} \angle 45°$
$i_1(t)$
$-J4$
$J6$

$$i_1(t) = \frac{2}{\sqrt{2}} \angle 45° * \frac{2 + j6}{2 + j6 - j4}$$

2
J4
$i_2(t)$
$-J6$
$\frac{3}{\sqrt{2}} \angle 30°$

$$i_2(t) = \frac{\frac{3}{\sqrt{2}} \angle 30°}{2 + J4 - J6}$$

$$i(t) = i_1(t) + i_2(t).$$

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متروك

للطالب

متروك
للطالب